# MOTION OF A RIGID STRIP-MASS SOLDERED INTO AN ELASTIC MEDIUM, EXCITED BY A PLANE WAVE

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The plane problem of motion of a rigid strip-mass of finite constant width and infinite length,which is in rigid contact with the infinite elastic medium and is acted upon by a plane elastic wave is reduced to two boundary value problems of the dynamic theory of elasticity for the half-space,which are solved by the Wiener-Hopf-Fok method.

We have obtained Formulas for the components of the displacement and angle of rotation of the strip which for finite intervals of time, contain a finite number of quadratures.

An analogous problem for a strip lying on the surface of the elastic half-space was considered in [1]. A related acoustic problem was investigated by an analogous method by Afanas'ev [2].

**1.** Consider a rigid strip-mass of infinite length and finite constant width in a rigid contact with the elastic medium which occupies the infinite space. Let us choose the units of length, time and mass in such a way that the half of the strip width, the density of the medium, and the velocity of the transverse waves become equal to unity. We introduce a Cartesian coordinate system $xyz$ and place the strip along the $z$-axis, so that $y = 0$, $-\infty < z < \infty$, and $|x| \leqslant 1$.

Let the strip be acted upon by a plane wave whose front is parallel to the edge of the strip (Fig. 1) and reaches it at the instant $t = 0$. Under the above assumptions all the quantities are independent of the cooredinate $z$, i.e. the medium is in the state of plane strain. When $t \leqslant 0$ the strip is at rest, and the total displacement vector of the medium with the components $u_n, v_n$ coincides with the displacement vector of the moving wave

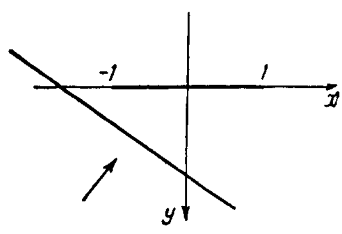


Fig. 1

$$u_n(x, y, t) = u_i(t - \vartheta(x+1) + \delta y)$$
$$v_n(x, y, t) = v_i(t - \vartheta(x+1) + \delta y) \qquad (1.1)$$
$$\text{for} \quad t \leqslant 0$$

The functions $u_i$ and $v_i$ satisfy conditions

$$u_i(\tau) = v_i(\tau) = 0 \quad \text{for } \tau \leqslant 0 \qquad (1.2)$$

If the type of the moving wave is fixed (longitudinal or transverse), the expressions relating the functions $u_i$ and $v_i$ as well as quantities $\vartheta$ and $\delta$ can be obtained. However, this will not be necessary in what follows. Let us note only that for thw motion of the transverse wave $\vartheta \leqslant 1$ and for the motion of the longitudinal wave $\vartheta \leqslant \gamma$, where $\gamma$ is the inverse value of the velocity of the longitudinal waves $(\gamma < 1)$.

For $t > 0$ the motion of the elastic medium is described by Expressions

$$u_n(x, y, t) = u_i(t - \vartheta(x+1) + \delta y) + u(x, y, t)$$
$$v_n(x, y, t) = v_i(t - \vartheta(x+1) + \delta y) + v(x, y, t) \qquad (1.3)$$

where $u$ and $v$ are components of the displacement in the disturbance caused by the presence of the strip. From (1.1) and (1.3) follow zero initial conditions for the functions $u$ and $v$.

When $t > 0$ the strip is set in motion. The objective of the present paper is to describe that motion. Since the strip is nondeformable its motion can be described by the displacement of the center of gravity with components $u_0(t)$ and $v_0(t)$ and the angle of rotation $\alpha(t)$. Equations of motion of the strip have the form

$$mu_0^{\cdot\cdot} = R_x(t), \qquad mv_0^{\cdot\cdot} = R_y(t), \qquad J\alpha^{\cdot\cdot} = M(t) \qquad (1.4)$$

Here $m$ is the mass, $J$ is the moment of inertia of a unit length of the strip, $R_x(t)$ and $R_y(t)$ are the components of the resultant force and $M(t)$ is the moment of forces acting on a unit length of the strip. Due to the rigid contact between the strip and the medium we have (1.5)

$$R_x(t) = \int_{-1}^{1} [\tau_{xy}]\, dx, \qquad R_y(t) = \int_{-1}^{1} [\sigma_{yy}]\, dx, \qquad M(t) = \int_{-1}^{1} [\sigma_{yy}](x - x_0)\, dx$$

where $x_0$ is the coordinate of the center of gravity of the strip in the position of equilibrium, and

$$[\tau_{xy}] = \tau_{xy}(x, +0, t) - \tau_{xy}(x, -0, y)$$
$$[\sigma_{yy}] = \sigma_{yy}(x, +0, t) - \sigma_{yy}(x, -0, t) \qquad (1.6)$$

are discontinuities of the stress components on the strip. If the strip were assumed to be homogeneous we should set $x_0 = 0$ and $J = \frac{1}{3}m$.

Equations (1.4) are insufficient for the description of motion of the strip, since their right-hand sides (1.5) in turn depend upon that motion.

In order to clarify the nature of this dependence we will assume for a while that the quantities $u_0(t)$, $v_0(t)$ and $\alpha(t)$ are given functions of time.

Then from the condition of rigid contact between the strip and the medium we obtain

$$u_n = u_0(t), \qquad v_n = v_0(t) + (x - x_0)\,\alpha(t) \qquad \text{for } y = 0,\ |x| \leqslant 1 \tag{1.7}$$

In conjunction with (1.3) this gives boundary conditions for $u$ and $v$

$$\begin{aligned} u &= u_0(t) - u_i(t - (x+1)\vartheta) \\ v &= v_0(t) - v_i(t - (x+1)\vartheta) + (x - x_0)\,\alpha(t) \end{aligned} \qquad \begin{pmatrix} y = 0 \\ |x| \leqslant 1 \end{pmatrix} \tag{1.8}$$

Equations of motion of the medium in our system of units have the form

$$\begin{aligned} \gamma^2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} - \gamma^2 \frac{\partial^2 u}{\partial y^2} - (1 - \gamma^2)\frac{\partial^2 v}{\partial x\,\partial y} = 0 \\ \gamma^2 \frac{\partial^2 v}{\partial t^2} - \gamma^2 \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} - (1 - \gamma^2)\frac{\partial^2 u}{\partial x\,\partial y} = 0 \end{aligned} \tag{1.9}$$

As is well known, these Equations are satisfied by Expressions

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{1.10}$$

where $\varphi(x,y,t)$ and $\psi(x,y,t)$ are the potentials of the longitudinal and transverse waves, which satisfy wave Equations

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \gamma^2 \frac{\partial^2 \varphi}{\partial t^2}, \qquad \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial^2 \psi}{\partial t^2} \tag{1.11}$$

Now, following Maue [2], we will represent the displacement vector $(u,v)$ as a sum of two components, the symmetric and the antisymmetric ones with respect to the plane $y = 0$

$$u = u_{(1)} + u_{(2)}, \qquad v = v_{(1)} + v_{(2)} \tag{1.12}$$

Here $u_{(1)}$, $v_{(2)}$, $\sigma_{(1)}$, $\tau_{(2)}$, $\varphi_{(1)}$ and $\psi_{(2)}$ are even, and $u_{(2)}$, $v_{(1)}$, $\sigma_{(2)}$, $\tau_{(1)}$, $\varphi_{(2)}$ and $\psi_{(1)}$ are odd functions of $y$. From here on we drop the subscripts $xy$ and $yy$ of stress components.

The displacements components must be continuous everywhere for $y = 0$ (as functions of $y$) and the stress components – outside of the strip. Hence it follows that

$$\begin{aligned} u_{(2)} = v_{(1)} = 0 \qquad \text{for } y = 0,\ -\infty < x < \infty \\ \tau_{(1)} = \sigma_{(2)} = 0 \qquad \text{for } y = 0,\ |x| > 1 \end{aligned} \tag{1.13}$$

In place of (1.6) we obtain

$$[\sigma_{yy}] = 2\sigma_{(2)}(x, +0, t), \qquad [\tau_{xy}] = 2\tau_{(1)}(x, +0, t) \tag{1.14}$$

The symmetric and the antisymmetric parts of the field each separately satisfy the equations of motion. Their boundary conditions are established independently and hence they are determined independently of each other. Moreover, it is sufficient to find them in the half-space $y \geqslant 0$ only, since they can be continued into the half-space $y < 0$ by means of their properties of evenness stated above. From (1.8) and (1.13) follow boundary conditions

$$u_{(1)} = u_0(t) - u_i(t - (x+1)\vartheta) \qquad \text{for } y=0,\ |x| \leqslant 1$$
$$v_{(1)} = 0 \quad \text{for } y=0,\ -\infty < x < \infty, \qquad \tau_{(1)} = 0 \quad \text{for } y=0,\ |x| > 1 \qquad (1.15)$$

$$v_{(2)} = v_0(t) + (x - x_0)\,\alpha\,(t - v_i(t) - (x+1)\vartheta) \qquad \text{for } y=0,\ |x| \leqslant 1$$
$$u_{(2)} = 0 \quad \text{for } y=0,\ -\infty < x < \infty, \qquad \sigma_{(2)} = 0 \quad \text{for } y=0,\ |x| > 1 \qquad (1.16)$$

In what follows we will also need Expressions of stresses in terms of potentials

$$\sigma_{(j)} = \left(\frac{\partial^2}{\partial t^2} - 2\frac{\partial^2}{\partial x^2}\right)\varphi_{(j)} - 2\frac{\partial^2}{\partial x \partial y}\psi_{(j)}, \qquad \tau_{(j)} = \left(\frac{\partial^2}{\partial t^2} - 2\frac{\partial^2}{\partial x^2}\right)\psi_{(j)} + 2\frac{\partial^2}{\partial x \partial y}\varphi_{(j)} \qquad (j = 1,2) \qquad (1.17)$$

Because the strip has sharp edges the boundary and initial conditions alone are insufficient for the determination of the unique solution of the problem. The additional condition, the so called "condition at the edge" (see, for instance, [3]) will be formulated as follows: we will require that in the vicinity of the edges the displacements be bounded, and the stresses increase not faster than the inverse value of the square root of the distance to the edge.

Thus we arrive at two independent boundary value problems for the half-space with mixed boundary conditions given on three parts of the boundary for the determination of the symmetric and the antisymmetric parts of the solution.

**2.** Let us perform the double Laplace transformation with respect to $x$ and $t$

$$\varphi(q, y, p) = \int_0^\infty dt \int_{-\infty}^{\infty} \exp(-qx - pt)\,\varphi(x, y, t)\,dx$$
$$\varphi(x, y, t) = \frac{-1}{4\pi^2}\int_{-i\infty+c}^{i\infty+c} dp \int_{-i\infty+c'}^{i\infty+c'} \exp(qx + pt)\,\varphi(q, y, p)\,dq \qquad (2.1)$$

where $c > 0$ and $c'$ is such that the path of integration with respect to $q$ lies in the region of regularity of $\varphi(q,y,p)$ as a function of $q$, and analogous transformations of all the other quantities. The transforms will be designated by the same letters as the originals, the only difference, whenever necessary, being shown in arguments. We will also consider single transformation with respect to $t$. For example

$$v_0(p) = \int_0^\infty e^{-pt} v_0(t)\,dt, \qquad v_0(t) = \frac{1}{2\pi i}\int_{-i\infty+c}^{i\infty+c} e^{pt} v_0(p)\,dp \qquad (2.2)$$

Applying transformations (2.1) to wave Equations (1.11) we obtain

$$\frac{\partial^2\varphi(q,y,p)}{\partial y^2}-(\gamma^2p^2-q^2)\varphi(q,y,p)=0$$
$$\frac{\partial^2\psi(q,y,p)}{\partial y^2}-(p^2-q^2)\psi(q,y,p)=0 \qquad (2.3)$$

Solutions of these Equations, compatible with zero initial conditions, should be chosen in the form

$$\begin{aligned}\varphi(q,y,p)&=\varphi(q,p)\exp\left(-\sqrt{\gamma^2p^2-q^2}\,y\right)\\ \psi(q,y,p)&=\psi(q,p)\exp\left(-\sqrt{p^2-q^2}\,y\right)\end{aligned}\qquad (y>0)\qquad (2.4)$$

The roots contained in above Expressions should be presented in a form so as to have

$$\operatorname{Re}\sqrt{\gamma^2p^2-q^2}\geqslant 0,\qquad \operatorname{Re}\sqrt{p^2-q^2}\geqslant 0 \qquad (2.5)$$

and it is sufficient that these conditions be fulfilled in the band $-\gamma\operatorname{Re}p<\operatorname{Re}q<\gamma\operatorname{Re}p$. To that end in the complex plane $q$ we draw the branch cuts along the rays $q=\pm ps$, where $s$ is real and $\gamma\leqslant s<\infty$.

The transforms of the boundary values of displacements and stresses can be presented in the form

$$\begin{aligned}u_{(j)}(q,0,p)&=u_{(j)}^+(q,p)\,e^{-q}+u_{(j)}^-(q,p)\,e^{q}+u_{(j)}^\circ(q,p)\\ v_{(j)}(q,0,p)&=v_{(j)}^+(q,p)\,e^{-q}+v_{(j)}^-(q,p)\,e^{q}+v_{(j)}^\circ(q,p)\\ \tau_{(j)}(q,0,p)&=\tau_{(j)}^+(q,p)\,e^{-q}+\tau_{(j)}^-(q,p)\,e^{q}+\tau_{(j)}^\circ(q,p)\\ \sigma_{(j)}(q,0,p)&=\sigma_{(j)}^+(q,p)\,e^{-q}+\sigma_{(j)}^-(q,p)\,e^{q}+\sigma_{(j)}^\circ(q,p)\qquad (j=1,2)\end{aligned}\qquad (2.6)$$

$$u_{(j)}^+(q,p)=\int_0^\infty e^{-q\xi}u_{(j)}(\xi+1;0,p)\,d\xi,\qquad u_{(j)}^\circ(q,p)=\int_{-1}^{1}e^{-qx}u_{(j)}(x,0,p)\,dx$$
$$u_{(j)}^-(q,p)=\int_0^\infty e^{q\xi}u_{(j)}(-\xi-1,0,p)\,d\xi \qquad (2.7)$$

and analogously for the rest of quantities in (2.6).

From boundary conditions (1.15) and (1.16) it follows that

$$\begin{aligned}v_{(1)}(q,0,p)&=\tau_{(1)}^+(q,p)=\tau_{(1)}^-(q,p)=0\\ u_{(2)}(q,0,p)&=\sigma_{(2)}^+(q,p)=\sigma_{(2)}^-(q,p)=0\end{aligned}\qquad (2.8)$$

Moreover, from kinematic considerations it can be concluded that the boundary values of all the quantities vanish for $t<x\cdot\min(\vartheta,\gamma)$, $x>0$ and $t<-x\gamma$, $x<0$. Hence it follows that $u_{(j)}^+$ and $v_{(j)}^+$ are regular as functions of $q$ in the half-plane $\operatorname{Re}q>-\min(\vartheta,\gamma)\operatorname{Re}p$, and $u_{(j)}^-$ and $v_{(j)}^-$ in the half-plane $\operatorname{Re}q<\gamma\operatorname{Re}p$. The quantities $\sigma_{(j)}^\circ$, $\tau_{(j)}^\circ$, $u_{(j)}^\circ$ and $v_{(j)}^\circ$, obviously are integer functions of $q$.

From the condition of the edge we obtain in the usual manner (see [4])

$$u_{(1)}^{+}(q,p)=O(|q|^{-1}),\qquad v_{(2)}^{+}(q,p)=O(|q|^{-1}) \tag{2.9}$$
$$e^{-q}\sigma_{(2)}^{\circ}(q,p)=O(|q|^{-1/2}),\qquad e^{-q}\tau_{(1)}^{\circ}(q,p)=O(|q|^{-1/2})\quad \text{for } |q|\to\infty\ \mathrm{Re}\, q>0$$
$$u_{(1)}^{-}(q,p)=O(|q|^{-1}),\qquad v_{(2)}^{-}(q,p)=O(|q|^{-1})$$
$$e^{q}\sigma_{(2)}^{\circ}(q,p)=O(|q|^{-1/2}),\qquad e^{q}\tau_{(1)}^{\circ}(q,p)=O(|q|^{-1/2})\quad \text{for } |q|\to\infty\ \mathrm{Re}\, q<0$$

Furthermore, it follows from boundary conditions (1.15) and (1.16) that

$$u_{(1)}^{\circ}(q,p)=-\frac{2u_i(p)\sinh(q+\vartheta p)\,e^{-\vartheta p}}{q+\vartheta p}+\frac{2\sinh q\, u_0(p)}{q}$$
$$v_{(2)}^{\circ}(q,p)=-\frac{2v_i(p)\sinh(q+\vartheta p)\,e^{-\vartheta p}}{q+\vartheta p}+ \tag{2.10}$$
$$+\frac{2\sinh q\,[v_0(p)-x_0\alpha(p)]}{q}+2\left(\frac{\sinh q}{q^2}-\frac{\cosh q}{q}\right)\alpha(p)$$

From (1.10) and (1.17) we have

$$u_{(j)}(q,y;p)=q\varphi_{(j)}(q,y,p)-\sqrt{p^2-q^2}\,\psi_{(j)}(q,y,p) \tag{2.11}$$
$$v_{(j)}(q,y,p)=-\sqrt{\gamma^2p^2-q^2}\,\varphi_{(j)}(q,y,p)-q\psi_{(j)}(q,y,p)\qquad (j=1,2)$$
$$\tau_{(j)}(q,y,p)=-2q\sqrt{\gamma^2p^2-q^2}\,\varphi_{(j)}(q,y,p)+(p^2+2q^2)\,\psi_{(j)}(q,y,p)$$
$$\sigma_{(j)}(q,y,p)=(p^2-2q^2)\,\varphi_{(j)}(q,y,p)+2q\sqrt{p^2-q^2}\,\psi_{(j)}(q,y,p)$$

Hence by means of (2.4), (2.6), (2.8) and (2.10) we obtain

$$\tau_{(1)}^{\circ}(q,p)+\frac{p^2\sqrt{\gamma^2p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q^2}}\,[e^{q}u_{(1)}^{-}(q,p)+e^{-q}u_{(1)}^{+}(q,p)]=$$
$$=\frac{2p^2\sqrt{\gamma^2p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q^2}}\left[\frac{u_i(p)\sinh(q+\vartheta p)\,e^{-p\vartheta}}{q+\vartheta p}-\frac{u_0(p)\sinh q}{q}\right] \tag{2.12}$$

$$\sigma_{(2)}^{\circ}(q,p)+\frac{p^2\sqrt{p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q^2}}\,[e^{q}v_{(2)}^{-}(q,p)+e^{-q}v_{(2)}^{+}(q,p)]=$$
$$=\frac{2p^2\sqrt{p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q}}\left[\frac{v_i(p)\sinh(q+\vartheta p)\,e^{-p\vartheta}}{q+\vartheta p}-\right. \tag{2.13}$$
$$\left.-\frac{\sinh q\,[v_0(p)-x_0\alpha(p)]}{q}+\left(\frac{\cosh q}{q}-\frac{\sinh q}{q^2}\right)\alpha(p)\right]$$

Equations (2.12) and (2.13) are the generalized functional equations of Wiener-Hopf type. Their theory was considered in [4] (pp. 222 - 224). Let us introduce two following functions of the complex variable $s$: $K_{(1)}(s)$ and $K_{(2)}(s)$. They are regular and having no zeros in the plane $s$ which is cut along the segment of the real axis $\gamma\leqslant s<\infty$, and they are such that

$$pK_{(1)}\left(\frac{q}{p}\right)K_{(1)}\left(-\frac{q}{p}\right)=\frac{p^2\sqrt{\gamma^2p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q^2}} \tag{2.14}$$
$$pK_{(2)}\left(\frac{q}{p}\right)K_{(2)}\left(-\frac{q}{p}\right)=\frac{p^2\sqrt{p^2-q^2}}{q^2+\sqrt{\gamma^2p^2-q^2}\,\sqrt{p^2-q^2}} \tag{2.15}$$

These functions are (see [2 and 5])

$$K_{(1)}(s) = \sqrt{\frac{2(\gamma - s)}{1+\gamma^2}}\, e^{g(s)}, \qquad K_{(2)}(s) = \sqrt{\frac{2(1-s)}{1+\gamma^2}}\, e^{g(s)} \tag{2.16}$$

where

$$g(s) = \frac{1}{\pi}\int_{\gamma}^{1} \tan^{-1}\sqrt{(1-\gamma^2\zeta^{-2})(\zeta^{-2}-1)}\,\frac{d\zeta}{\zeta - s} \tag{2.17}$$

moreover, if $s$ is real and lies in the interval $(\gamma, 1)$, the integral is interpreted in the sense of principal value. When $s$ is real we will also use the following designations:

$$K_{(j)}(s+i0) = M_{(j)}(s) - iN_{(j)}(s) = M_{(j)}(s) - iK_{(j)}(-s)\,L_{(j)}(s) \tag{2.18}$$

It is easy to see that for $s < \gamma$

$$M_{(j)}(s) = K_{(j)}(s), \qquad N_{(j)}(s) = L_{(j)}(s) = 0 \tag{2.19}$$

Furthermore, in what follows we will need coefficients of the Taylor series expansion of $K_{(j)}(s)$ in the neighborhood of point zero

$$K_{(j)}(s) = \sum_{n=0}^{\infty} k_{(j)\,n}\frac{s^n}{n!} \tag{2.20}$$

Thus, the problem has been reduced to the solution of Equations (2.12) and (2.13) in which the unknown functions must possess the analytic properties stated above.

3. First of all let us solve Equation (2.13). The solution of Equation (2.12) will then be obtained by means of a substitution

$$K_{(1)}(s) \to K_{(2)}(s), \qquad v_i(t) \to u_i(t), \qquad \alpha(t) \equiv 0$$

Multiplying both sides of (2.13) by $[K_{(2)}(q/p)]^{-1}e^q$, we can (see [4]) make use of analytic properties of functions contained in it and thus obtain the relation

$$v_{(2)}^{+}(q,p) - \frac{1}{2\pi i}\left[K_{(2)}\left(-\frac{q}{p}\right)\right]^{-1}\int_{-i\infty+c}^{i\infty+c}\frac{\exp(2q')}{q'-q}K_{(2)}\left(\frac{-q'}{p}\right)v_{(2)}^{-}(q',p)\,dq' =$$
$$= -\frac{\exp(-2p\vartheta)}{q+\vartheta p}v_i(p) + \frac{v_0(p)+(1-x_0)\alpha(p)}{q} + \frac{\alpha(p)}{q^2} +$$
$$+\frac{1}{2\pi i}\left[K_{(2)}\left(-\frac{q}{p}\right)\right]^{-1}\int_{-i\infty+c}^{i\infty+c}\frac{\exp(2q')}{q'-q}\left\{\frac{-v_i(p)}{q'+\vartheta p} +\right.$$
$$\left. + \frac{v_0(p)-(1+x_0)\alpha(p)}{q'} + \frac{\alpha(p)}{q'^2}\right\}K_{(2)}\left(-\frac{q'}{p}\right)dq' \qquad (0<c<\operatorname{Re} q<\infty) \tag{3.1}$$

In exactly the same way, multiplying (2.13) by $[K_{(2)}(-q/p)]^{-1}e^{-q}$, we obtain

$$v_{(2)}\ (q,\ p)+\frac{1}{2\pi i}\left[K_{(2)}\left(\frac{q}{p}\right)\right]^{-1}\int\limits_{-i\infty+c'}^{i\infty+c'}\frac{\exp(-2q')}{q'-q}K_{(2)}\left(\frac{q'}{p}\right)v_{(2)}{}^{+}(q',p)\,dq'=$$
$$=\frac{v_i(p)}{q+\vartheta p}\left[\frac{K_{(2)}(q/p)-K_{(2)}(-\vartheta)}{K_{(2)}(q/p)}\right]+\frac{(1+x_0)\,\alpha(p)-v_0(p)}{q}-\frac{\alpha(p)}{q^2}+$$
$$+\frac{k_{(2)\,0}\,[v_0(p)-(1+x_0)\,\alpha(p)]+p^{-1}k_{(2)\,1}\alpha(p)}{qK_{(2)}(q/p)}+\frac{k_{(2)\,0}\alpha(p)}{q^2K_{(2)}(q/p)}-$$
$$-\frac{1}{2\pi i}\left[K_{(2)}\left(\frac{q}{p}\right)\right]^{-1}\int\limits_{-i\infty+c'}^{i\infty+c'}\frac{\exp(-2q')}{q'-q}\left\{\frac{\exp(-2p\vartheta)}{q'+p\vartheta}v_i(p)-\right.$$
$$\left.-\frac{v_0(p)+(1-x_0)\,\alpha(p)}{q'}-\frac{\alpha(p)}{q'^2}\right\}K_{(2)}\left(\frac{q'}{p}\right)dq'\qquad(-\infty<\operatorname{Re}q<c'<\gamma)\quad(3.2)$$

In (3.1) let us replace $q'$ by $-q'$. The integrals contained in the resulting equation and in Equation (3.2) can be reduced to integrals along the branch cuts by deforming the contours of integration on the right-hand half-plane $q'$. In the equation obtained from (3.1) we set $q = ps$ and $q'=p\zeta$, and in that obtained from (3.2) $q = -ps$ and $q' = p\zeta$, and thus form a system of two Fredholm's integral equations which is decomposed into two independent equations with respect to the sum and the difference of the unknown functions. Integrating these equations we obtain the formal solution in terms of the Neumann series with the aid of which we easily find Expressions for $v_{(2)}{}^{+}(p,\ q)$ and $v_{(2)}{}^{-}(p,\ q)$ and from (2.13) we obtain

$$\sigma_{(2)}{}^{\circ}(q,\ p)\equiv\sigma_{(2)}(q,\ 0,\ p)=-K_{(2)}\left(\frac{q}{p}\right)e^{-q}\times$$
$$\times\left\{\sum_{k=1}^{\infty}\int\limits_{\gamma}^{\infty}\cdots\int\limits_{\gamma}^{\infty}\Pi_{(2)\,k}(\zeta_1,\ \zeta_2,\ \ldots,\ \zeta_k)\,F_{(2)\,k}(\zeta_1;\ p,\ \vartheta)\,\frac{d\zeta_1\ldots d\zeta_k}{\zeta_k-qp^{-1}}+\right.$$
$$\left.+F_{(2)\,0}\left(\frac{q}{p},\ p,\ \vartheta\right)\right\}-K_{(2)}\left(-\frac{q}{p}\right)e^{q}\times$$
$$\times\left\{\sum_{k=1}^{\infty}\int\limits_{\gamma}^{\infty}\cdots\int\limits_{\gamma}^{\infty}\Pi_{(2)\,k}(\zeta_1,\zeta_2,\ldots,\zeta_k)\,F_{(2)\,k+1}(\zeta_1,\ p,\ \vartheta)\,\frac{d\zeta_1\ldots d\zeta_k}{\zeta_k+qp^{-1}}+\right.$$
$$\left.+F_{(2)\,1}\left(-\frac{q}{p},\ p,\ \vartheta\right)\right\}\qquad(3.3)$$

Here

$$\Pi_{(2)\,k}=\frac{L_{(2)}(\zeta_1)}{\pi^k}\prod_{j=2}^{k}\frac{L_{(2)}(\zeta_j)}{\zeta_j+\zeta_{j-1}}\exp\left(-2p\sum_{l=1}^{k}\zeta_l\right)\qquad(k=2,3,\ldots)\qquad(3.4)$$
$$\Pi_{(2)\,1}=\frac{L_{(2)}(\zeta_1)}{\pi}\exp(-2p\zeta_1)$$

$$F_{(2)k}(\zeta, p, \vartheta) = \frac{M_{(2)}((-1)^k\vartheta)\exp\{-p\vartheta[1+(-1)^k]\}v_i(p)}{\zeta+(-1)^k\vartheta} - \tag{3.4}$$

$$-\frac{k_{(2)0}[v_0(p) - x_0\alpha(p) + (-1)^k\alpha(p)]}{\zeta} + \frac{(-1)^k k_{(2)1}\alpha(p)}{p\zeta} - \frac{(-1)^k k_{(2)0}\alpha(p)}{p\zeta^2}$$

The investigation of convergence of the series in (3.3) is difficult, however, by performing the inverse Laplace transformations we will find that for finite intervals of time the originals corresponding to all the terms of those series starting with some number (which depends on $t$) vanish, i.e. the series becomes a finite sum. It can be verified that the sum of the remaining terms in Expression for $\sigma_{(2)}(x, 0, t)$ and in corresponding Expressions for $v_{(2)}(x, 0, t)$ satisfies the boundary conditions.

In order to obtain Expression for $\tau_{(1)}{}^{\circ}(p, q)$, one has to replace $K_{(2)}$ by $K_{(1)}$ and $L_{(2)}$ by $L_{(1)}$ in (3.3) and (3.4) and set

$$\alpha(p) \equiv 0, \quad v_i(p) \equiv u_i(p), \quad v_0(p) \equiv u_0(p)$$

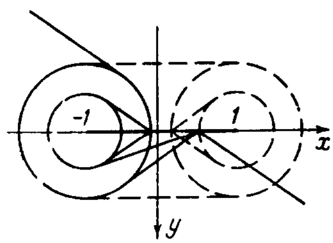

Fig.2

4. It follows from (1.4), (1.14), (2.1) and (2.2) that

$$R_x(p) = 2\lim_{q\to 0}\tau_{(1)}{}^{\circ}(q, p)$$

$$R_y(p) = 2\lim_{q\to 0}\sigma_{(2)}{}^{\circ}(q, p) \tag{4.1}$$

$$M(p) = -2\lim_{q\to 0}\frac{\partial}{\partial q}\sigma_{(2)}{}^{\circ}(q, p) - R_y(p)\,x_0$$

From the relation (3.3) we easily obtain

$$R_x(p) = -2k_{(1)0}\times \tag{4.2}$$

$$\times\left\{\sum_{k=1}^{\infty}\int_{\gamma}^{\infty}\cdots\int_{\gamma}^{\infty}\Pi_{(1)k}(\zeta_1,\ldots,\zeta_k)\frac{F_{(1)0}(\zeta_1, p, \vartheta) + F_{(1)1}(\zeta_1, p, \vartheta)}{\zeta_k}d\zeta_1\ldots d\zeta_k + \right.$$

$$\left. + \frac{u_i(p)M_{(1)}(\lambda)\exp[-p(\vartheta+\lambda)]}{\vartheta}\Bigg|_{\lambda=-\vartheta}^{\lambda=\vartheta} + 2(pk_{(1)0} - k_{(1)1})u_0(p)\right\}$$

$$R_y(p) = -2k_{(2)0}\times \tag{4.3}$$

$$\times\left\{\sum_{k=1}^{\infty}\int_{\gamma}^{\infty}\cdots\int_{\gamma}^{\infty}\Pi_{(2)k}(\zeta_1,\ldots,\zeta_k)\frac{F_{(2)0}(\zeta_1, p, \vartheta) + F_{(2)1}(\zeta_1, p, \vartheta)}{\zeta_k}d\zeta_1\ldots d\zeta_k + \right.$$

$$\left. + \frac{v_i(p)M_{(2)}(\lambda)\exp[-p(\vartheta+\lambda)]}{\vartheta}\Bigg|_{\lambda=-\vartheta}^{\lambda=\vartheta} + 2(pk_{(2)0} - k_{(2)1})[v_0(p) - x_0\alpha(p)]\right\}$$

$$M(p) = -2p^{-1}k_{(2)0} \times \tag{4.4}$$

$$\times \sum_{k=1}^{\infty}(-1)^k \int_{\gamma}^{\infty}\ldots\int_{\gamma}^{\infty}\Pi_{(2)k}(\zeta_1,\ldots,\zeta_k)\,\frac{F_{(2)\,0}(\zeta_1,p,\vartheta)-F_{(2)\,1}(\zeta_1,p,\vartheta)}{\zeta_k}\,d\zeta_1\ldots d\zeta_k +$$

$$+\frac{2v_i(p)\,[\lambda k_{(2)\,1}-(1+\lambda p)\,k_{(2)\,0}\,]\,M_{(2)}(\lambda)}{p\vartheta^2}\,e^{-p\,(\vartheta+\lambda)}\Big|_{\lambda=-\vartheta}^{\lambda=\vartheta}+$$

$$+\frac{4\,({}^3/_2 k_{(2)\,1}\,k_{(2)\,2}-{}^1/_2 k_{(2)\,0}\,k_{(2)\,3}-3pk^2_{(2)\,1}+3p^2k_{(2)\,0}\,k_{(2)\,1}-p^3k_{(2)\,0}^{\,2})\,\alpha\,(p)}{3p^2}-x_0R_v(p)$$

Applying the Laplace transformation with respect to time to Equations (1.4) and substituting the resulting equations into Expressions (4.2),(4.3) and (4.4), by the rules of the operational calculus we find Expressions for $u_0(t)$, $v_0(t)$ and $\alpha(t)$, which after some transformations can be presented in the form

$$u_0(t) = \sum_{j=1}^{2} a_j e^{\mu_j t} * R_{(1)}(t) \tag{4.5}$$

$$v_0(t) = \sum_{j=1}^{6} e^{\nu_j t} * [b_j R_{(2)}(t) + x_0 c_j M_j(t)] \tag{4.6}$$

$$\alpha(t) = \sum_{j=1}^{6} e^{\nu_j t} * [x_0 d_j R_{(2)}(t) + e_j M_j(t)] \tag{4.7}$$

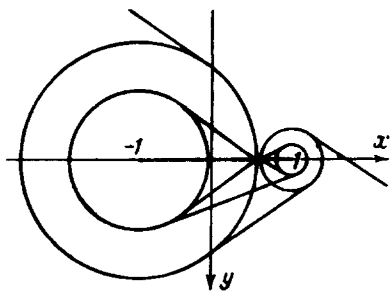


Fig. 3

Here the mark * designates the convolution

$$f(t) * g(t) = \int_0^t f(t-\tau)\,g(\tau)\,d\tau \tag{4.8}$$

The quantities $R_{(1)}(t)$, $R_{(2)}(t)$ and $M_j(t)$ are determined by Expressions

$$R_{(1)} = -\frac{k_{(1)0}\,[M_{(1)}(\lambda)\,\delta(t)+{}^1/_2\,\lambda k_{(1)0}\,S_{(1)11}(t-2\gamma,\lambda)]}{\vartheta} * u_j(t-\vartheta-\lambda)\Big|_{\lambda=-\vartheta}^{\lambda=\vartheta} +$$

$$+ k^2_{(1)0} S_{(1)\,11}(t,0) * u_0(t-2\gamma) \tag{4.9}$$

$$R_{(2)} = -\frac{k_{(2)\,0}\,[M_{(2)}(\lambda)\,\delta(t)+{}^1/_2\,\lambda k_{(2)0}\,S_{(2)\,11}(t-2\gamma,\lambda)]}{\vartheta} * v_i(t-\vartheta-\lambda)\Big|_{\lambda=-\vartheta}^{\lambda=\vartheta} +$$

$$+ k_{(2)\,0}^{\,2}[v_0(t-2\gamma)-x_0\alpha(t-2\gamma)] * S_{(2)\,11}(t,0) \tag{4.10}$$

$$M_j = \vartheta^{-2}\nu_j\Big\{M_{(2)}(\lambda)\,[\lambda k_{(2)1}-(1+\nu_j)\,k_{(2)0}\,]\,\delta(t)+$$

$$+\frac{1}{2}\nu_j k^2_{(2)0} S_{(2)\,12}(t-2\gamma,\lambda)\Big\} * v_i(t-\vartheta-\lambda)\Big|_{\lambda=-\vartheta}^{\lambda=\vartheta} -$$

$$-\nu_j k_{(2)\,0}\,[(k_{(2)\,0}\,\nu_j - k_{(2)\,1})\,S_{(2)\,12}(t,0)+k_{(2)0}S_{(2)\,22}(t,0)] * \alpha(t-2\gamma) \tag{4.11}$$

The functions $S_{(j)\,ln}(t,\lambda)$ are equal to

$$S_{(j)\,ln}(t,\lambda)=\sum_{k=0}^{\infty}(-1)^{(n-1)\,k}S_{(j)\,k\,ln}(t-2k\gamma,\lambda) \qquad (j=1,2) \quad (4.12)$$

where

$$S_{(j)\,k\,ln}(t,\lambda)= \qquad (4.13)$$

$$=\int_0^{1/2t}d\tau_k\int_0^{\tau_k}d\tau_{k-1}\cdots\int_0^{\tau_2}P_{(j)\,k}(\tau_1,\tau_2,\ldots,t)(\tau_1+\gamma+\lambda)^{-l}(1/2t-\tau_k+\gamma)^{-n}d\tau_1$$

$$(k\neq 0,1)$$

$$P_{(j)\,k}(\tau_1,\tau_2,\ldots,t)=\frac{L_{(j)}(\tau_1+\gamma)L_{(j)}(1/2t-\tau_k+\gamma)}{\pi^{k+1}(1/2t+\tau_{k-1}+2\gamma)}\prod_{l=2}^{k}\frac{L_{(j)}(\tau_l-\tau_{l-1}+\gamma)}{(\tau_l-\tau_{l-1}+2\gamma)} \qquad (4.14)$$

$$P_{(j)1}(\tau_1,t)=\frac{L_{(j)}(\tau_1+\gamma)L_{(j)}(1/2t-\tau_1+\gamma)}{\pi^2(1/2t+2\gamma)}$$

$$S_{(j)0\,ln}=\frac{L_{(j)}(1/2t+\gamma)}{\pi(1/2t+\gamma+\lambda)^l(1/2t+\gamma)^n} \qquad (4.15)$$

Moreover, we set

$$S_{(j)\,k\,ln}(\tau,\lambda)\equiv 0 \qquad \text{for } \tau<0 \qquad (4.16)$$

The quantities $\mu_j$ and $\nu_j$ are the roots of characteristic Equations

$$\Delta_{(1)}(\mu)\equiv m\mu^2+A_{(1)}\mu+B_{(1)}=0 \qquad (4.17)$$

$$\Delta_{(2)}(\nu)\equiv\Delta_{(2)1}(\nu)\,\Delta_{(2)2}(\nu)-x_0^{\,2}(A_{(2)}\nu+B_{(2)})^2\nu^2=0 \qquad (4.18)$$

Here

$$\Delta_{(2)1}\equiv m\nu^2+A_{(2)}\nu+B_{(2)} \qquad (4.19)$$

$$\Delta_{(2)2}\equiv J\nu^4+(1/3+x_0^{\,2})A_{(2)}\nu^3+B_{(2)}\nu^2(1+x_0^{\,2})+C_{(2)}\nu+D_{(2)}$$

$$A_{(j)}=4k_{(j)\,0}^{\,2},\qquad B_{(j)}=-4k_{(j)\,0}k_{(j)\,1} \qquad (j=1,2)$$

$$C_{(2)}=4k_{(2)\,1}^{\,2},\qquad D_{(2)}=2(1/3\,k_{(2)\,0}k_{(2)\,3}-k_{(2)\,1}k_{(2)\,2})$$

The coefficients $a_j, b_j, c_j, d_j$ and $e_j$ are defined by following Formulas:

$$a_j=\frac{2}{\Delta'_{(1)}(\mu_j)},\qquad b_j=\frac{2\Delta_{(2)\,2}(\nu_j)}{\Delta'_{(2)}(\nu_j)}$$

$$c_j=\frac{2\vartheta_j(A_{(2)}\nu_j+B_{(2)})}{\Delta_{(2)}(\nu_j)} \qquad (4.20)$$

$$d_j=-\frac{2m\nu_j^{\,4}}{\Delta'_{(2)}(\nu_j)},\qquad e_j=\frac{2\Delta_{(2)1}(\nu_j)}{\Delta'_{(2)}(\nu_j)}$$

where the stroke designates a derivative.

Expressions (3.5) to (4.7) represent the recurrence relations determining $u_0(t)$,

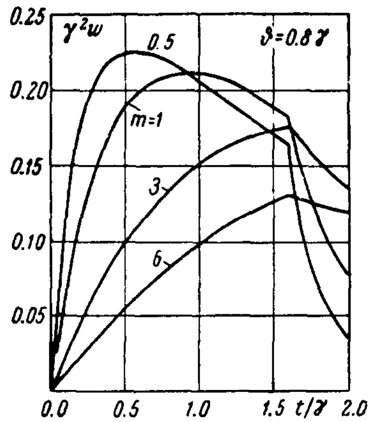


Fig. 4

$v_0(t)$ and $\alpha(t)$ for any $t$, since their right-hand parts contain the values of the unknown functions of the argument which is lagging by not less than $2\gamma$. Due to (4.16) Expressions (4.12) are finite sums with the number of terms (related to the waves diffracted a great number of times at the edges of the strip) which depends on the time and is equal to the integer part of $t/2\gamma$.

**5.** For the numerical interpretation of the obtained solution it is necessary first of all to compute the functions $K_{(j)}(-s)$, $M_{(j)}(s)$, $N_{(j)}(s)$ for a real and positive $s$. All those functions can be expressed in terms of the function $g(-s)$ by means of elementary operations. The latter is not expressed in terms of tabulated functions. The results of computations of $g(-s)$ according Formula (2.17) by means of numerical integration with the accuracy up to $10^{-4}$ for the case $\gamma = 1/\sqrt{3}$ are given in Table 1; we also give the values of constants

$$K_{(1)0} = 1.000,\quad K_{(1)1} = -0.7688,\quad K_{(1)2} = -0.6410,\quad K_{(1)3} = -0.6828$$
$$K_{(2)0} = 1.3161,\quad K_{(2)1} = -0.5301,\quad K_{(2)2} = -0.0921,\quad K_{(2)3} = 0.4497$$

For $t < 2\gamma$, when only once-diffracted waves are present, Formulas (4.9) to (4.11) take a particularly simple form. Here we should distinguish two following cases: $t < 2\vartheta$ and $2\vartheta < t < 2\gamma$. In the first case the front of the advancing wave has not yet reached the right-hand edge of the strip. The position of the wave fronts formed by this time is shown in Fig. 2. The dotted lines indicate the wave fronts formed due to the motion of the strip.

TABLE 1.

| $s\gamma^{-1}$ | $g(-s)$ | $s\gamma^{-1}$ | $g(-s)$ | $s\gamma^{-1}$ | $g(-s)$ |
|---|---|---|---|---|---|
| 0.0 | 0.0719 | 2.0 | 0.0282 | 4.0 | 0.0176 |
| | | 2.1 | 0.0274 | 4.1 | 0.0173 |
| 0.2 | 0.0622 | 2.2 | 0.0266 | 4.2 | 0.0170 |
| | | 2.3 | 0.0259 | 4.3 | 0.0167 |
| 0.4 | 0.0548 | 2.4 | 0.0252 | 4.4 | 0.0164 |
| | | 2.5 | 0.0245 | 4.5 | 0.0161 |
| 0.6 | 0.0490 | 2.6 | 0.0239 | 4.6 | 0.0158 |
| | | 2.7 | 0.0233 | 4.7 | 0.0155 |
| 0.8 | 0.0444 | 2.8 | 0.0227 | 4.8 | 0.0153 |
| | | 2.9 | 0.0222 | 4.9 | 0.0150 |
| 1.0 | 0.0405 | 3.0 | 0.0217 | 5.0 | 0.0148 |
| 1.1 | 0.0388 | 3.1 | 0.0212 | 5.1 | 0.0145 |
| 1.2 | 0.0373 | 3.2 | 0.0207 | 5.2 | 0.0143 |
| 1.3 | 0.0358 | 3.3 | 0.0202 | 5.3 | 0.0141 |
| 1.4 | 0.0345 | 3.4 | 0.0198 | 5.4 | 0.0139 |
| 1.5 | 0.0333 | 3.5 | 0.0194 | 5.5 | 0.0137 |
| 1.6 | 0.0321 | 3.6 | 0.0190 | 5.6 | 0.0135 |
| 1.7 | 0.0311 | 3.7 | 0.0186 | 5.7 | 0.0133 |
| 1.8 | 0.0301 | 3.8 | 0.0183 | 5.8 | 0.0131 |
| 1.9 | 0.0291 | 3.9 | 0.0179 | 5.9 | 0.0129 |
| | | | | 6.0 | 0.0128 |

In this case Formulas (4.9) to (4.11) have the form

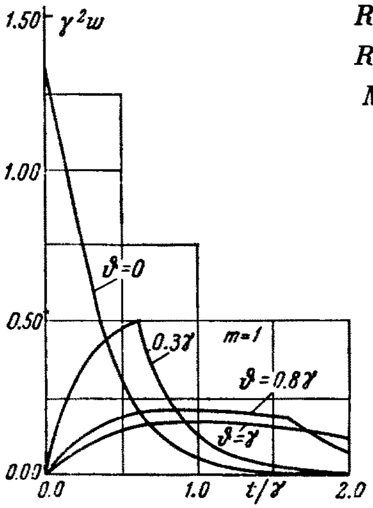


Fig. 5

$$R_{(1)}(t) = \vartheta^{-1} k_{(1)0} K_{(1)}(-\vartheta)\, u_i(t) \tag{5.1}$$

$$R_{(2)}(t) = \vartheta^{-1} k_{(2)0} K_{(2)}(-\vartheta)\, v_i(t) \tag{5.2}$$

$$M_j(t) = \vartheta^{-2} \nu_j \left[\vartheta k_{(2)1} + (1+\nu_j)\, k_{(2)0}\right] K_{(2)}(-\vartheta)\, v_i(t) \tag{5.3}$$

For $2\vartheta < t < 2\gamma$ there appear waves diffracted at the right-hand edge of the strip. The wave fronts formed by that time are shown in Fig. 3. For the sake of clarity of the sketch, the wave fronts related to the motion of the strip are omitted. Corresponding to waves diffracted at the right-hand edge of the strip, additional terms appear in Formulas (4.9) to (4.11) which then take form

$$R_{(1)}(t) = \vartheta^{-1} k_{(1)0} K_{(1)}(-\vartheta)\, u_i(t) - \vartheta^{-1} k_{(1)0} M_{(1)}(\vartheta)\, u_i(t-2\vartheta) \tag{5.4}$$

$$R_{(2)}(t) = \vartheta^{-1} k_{(2)0} K_{(2)}(-\vartheta)\, v_i(t) - \vartheta^{-1} k_{(2)0} M_{(2)}(\vartheta)\, v_i(t-2\vartheta) \tag{5.5}$$

$$\begin{aligned} M_j(t) &= \vartheta^{-2}\left[k_{(2)1}\vartheta + (1+\nu_j)\, k_{(2)0}\right] \nu_j K_{(2)}(-\vartheta)\, v_i(t) + \\ &+ v^{-2}\left[k_{(2)1}\vartheta - (1+\nu_j)\, k_{(2)0}\right] \nu_j M_{(2)}(\vartheta)\, v_i(t-2\vartheta) \end{aligned} \tag{5.6}$$

As an illustration we give the diagrams showing the dependence of the absolute value of the acceleration of the strip, equal to $\sqrt{\ddot{u}_0^2 + \ddot{v}_0^2}$ (the dots designate differentiation with respect to time), and its angular acceleration upon time for the case of a longitudinal attacking wave, in which the stress on the elementary areas parallel to the wave front is constant and equal to unity. In that case

$$u_i(t) = \vartheta t, \quad v_i(t) = -\sqrt{\gamma^2 - \vartheta^2}\, t \quad \text{for } t > 0 \tag{5.7}$$

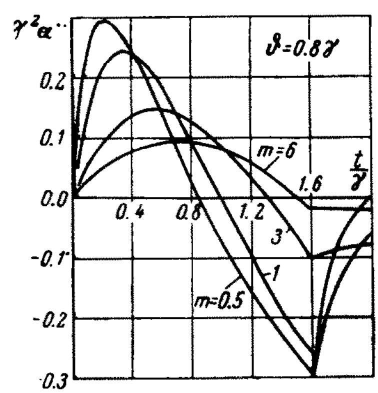


Fig. 6

Here we set $\gamma = 1/\sqrt{3}$ and assume the strip to be homogeneous. Fig. 4 illustrates the dependence of the value of acceleration $W = \sqrt{\ddot{u}_0^2 + \ddot{v}_0^2}$ upon time $t$ for the values of strip mass $m$ = = 0.5, 1.0, 3.0 and 6.0. First of all we note that for $t = 2\vartheta$ the curves have sharp turns related to the beginning of diffraction at the right-hand edge of the strip. Furthermore, the characteristic property of those curves for small values of the strip mass is the presence of a maximum which is shifted to the right-hand side as the mass increases and vanishes when the value of the mass is sufficiently high. Fig. 5 shows the dependence of the acceleration of the strip upon the angle of attack (the cosine of the angle of attack in our case equals $\vartheta\gamma^{-1}$) and time with the mass fixed. The strip attains the maximum acceleration when the wave is applied normally ($\vartheta = 0$) at the instant $t = 0$. For all other values of $\vartheta$ the acceleration equals zero initially and the larger the values of $\vartheta$ the smaller values it attains.

Fig. 6 shows the dependence of the angular acceleration of the strip upon time and mass.

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